Grammars and Semantics

- Programming languages are used to specify computations – that is, computations are the meaning/semantics of programs.

Reading

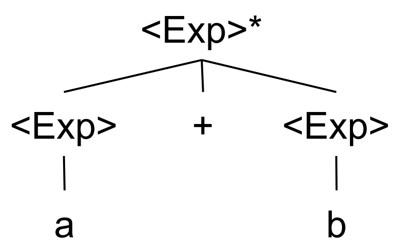
- Chap 3 in MPL

Grammars and Semantics

Consider the simple language of expressions:

```
G: <Exp>* ::= <Exp> + <Exp>
           |   <Exp> * <Exp>
           |   a
           |   b
           |   c
```

When we write the sentence `a + b` we can build the parse tree:

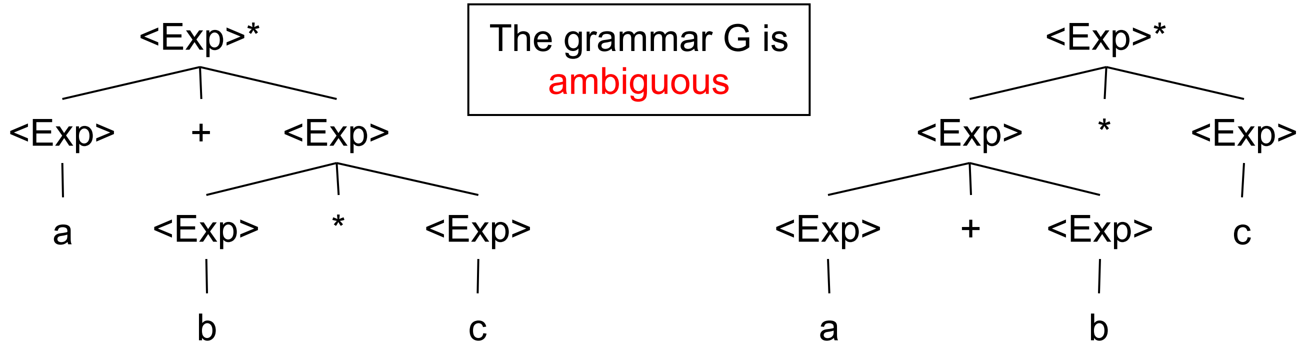


> We can say that this parse tree *represents* the computation `a + b`.

If we let a and b be variables, then the parse tree gives us a procedure to compute a + b by starting at the leaves of the tree: (1) lookup the values of the variables (2) pass the values up along the parse tree branches (3) use the values to compute the value of the + operator.

Grammars and Semantics

Now consider the sentence `a + b * c`, for this sentence we can construct two parse trees:



Even though both parse trees derive the same terminal string, the computations they represent are very different:

(1) left tree – first compute the product, then the addition
(2) right tree – first compute the addition, then the product

Since we had written the original sentence without parentheses the left parse tree represents the intended computation according to algebraic conventions.

However, from a machine point of view, there is no way of knowing which parse tree to pick…

Grammars and Semantics

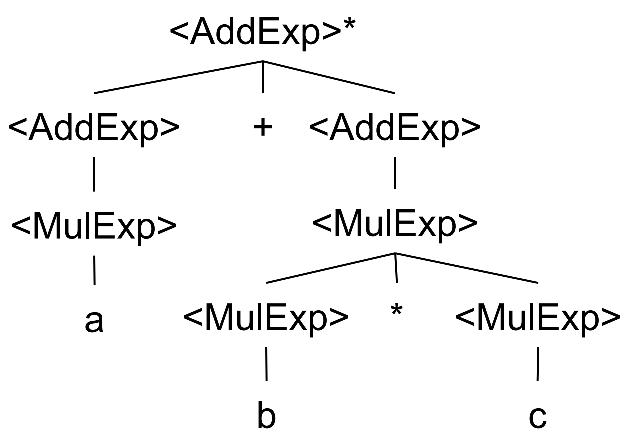
…we need additional information: operator precedence

Operator precedence means that some operators bind tighter than others, e.g. * binds tighter than +.

We can build operator precedence right into our grammar (“precedence climbing”):

```
G’: <AddExp>*::= <AddExp> + <AddExp>
             |   <MulExp>
    <MulExp> ::= <MulExp> * <MulExp>
             | a | b | c
```

Let’s try our problematic sentence `a + b * c`, only one parse tree is possible:



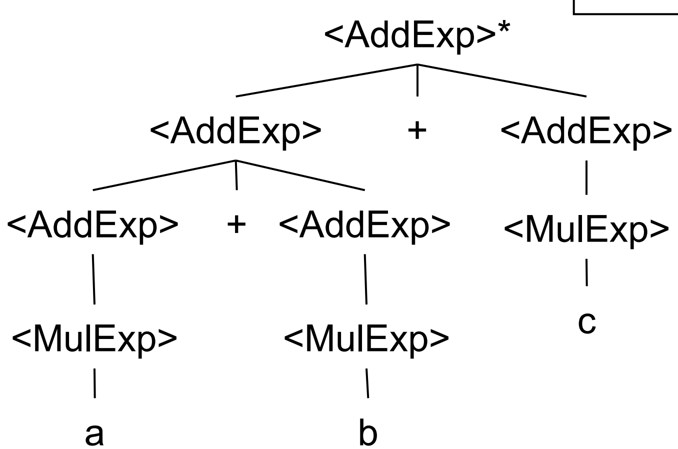
This is the only parse tree we can build, therefore, the grammar G’ is not ambiguous.

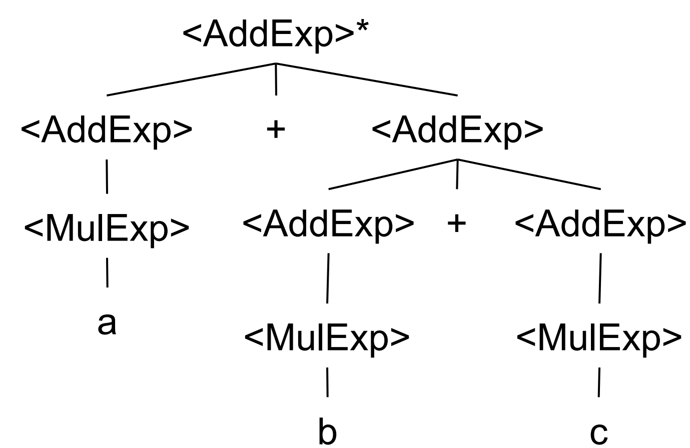
Grammars and Semantics

However, our new grammar still has a problem, consider the sentence a+b+c; here we have two possible parse trees:

```
G': <AddExp> ::= <AddExp> + <AddExp>
             |   <MulExp>
    <MulExp> ::= <MulExpr> * <MulExp>
             | a | b | c
```

The grammar G' is ambiguous





Grammars and Semantics

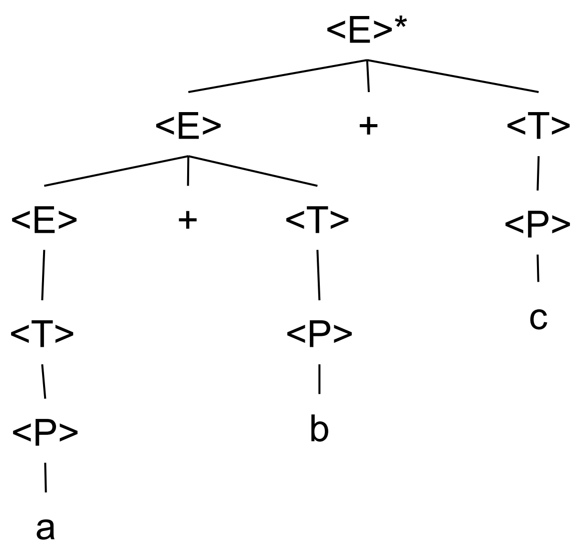
- Again, our grammar is ambiguous because the computation specified by the sentence a+b+c can be represented by two different parse trees.
- We need more information!
- There is one more algebraic property we have not yet explored – associativity
- Most algebraic operators, including the + operator, are left-associative.
- We can rewrite our grammar to take advantage of this additional information:

```
G”: <E>* ::= <E> + <T> | <T>
    <T>  ::= <T> * <P> | <P>
    <P>  ::= a | b | c
```

Grammars and Semantics

Let's try our sentence a+b+c again with grammar G'':

```
G”: <E>* ::= <E> + <T> | <T>
    <T>  ::= <T> * <P> | <P>
    <P>  ::= a | b | c
```



There is no other way to derive this string from the grammar and thus the grammar is not ambiguous.

Take Away

- Grammars can be ambiguous in the sense that a derived string can have multiple distinct parse trees.
- By taking additional information such as associativity and precedence about the operators of a language into account we can construct grammars that are not ambiguous.

Grammars and Semantics

Given the following grammar,

```
G”: <E>* ::= <E> + <T> | <T>
     <T>  ::= <T> * <P> | <P>
     <P>  ::= a | b | c
```

Add productions to the grammar that define the right-associative operator = at a lower precedence than any of the other operators.

This new operator should allow you to write expressions such as

a = b
a = b = c
a = b = b + c

Grammars and Semantics

a) Show that the following grammar is ambiguous.

```
G: <S> ::= <S> <S>
        |    ( <S> )
        |    ()
```

b) Rewrite the above grammar so that it is no longer ambiguous.

Class Exercise

- Let L(G) be the set of all strings that start with an a followed by zero or more b’s and end with the character c. Design grammar G.
- Given the following grammar Q:

```
Q: <A>*  ::= <A> @ <A>
           |    *
```

  - What are some of the strings in L(Q)?
  - Show that Q is ambiguous.

Assignment

- Assignment #7