A Type is a Set of Values

Consider the C statement:

int n = 3;

Here we constrain n to take on any value from the set of all integer values.

Reading: MPL Chap 6

Def: A type is a set of values.

Def: A <u>primitive type</u> is a type that is built into the language, e.g., integer, string.

Def: A <u>constructed type</u> is a user defined type, e.g., any type introduced by the user. In Asteroid this is done through the 'structure' statement.

Example: C, primitive type

float q;

type float ⇒ set of all possible floating point values

q is of type float, only a value that is a member of the set of all floating point values can be assigned to q.

Example: Java, constructed type

```
class Rectangle { int xdim; int ydim; };
```

Rectangle r = new Rectangle();

Now the variable r only accepts values that are members of type Rectangle; <u>object instantiations</u> of class Rectangle.

Example: Asteroid, constructed type

```
structure Rectangle with
  data xdim.
  data ydim.
end

let r:%Rectangle = Rectangle(4,2).
```

an element of type Rectangle.

In statically typed languages arrays are also considered 'constructed types'

Example: C, constructed type

the variable a will accept values which are arrays of 3 integers.

e.g.: int a[3] =
$$\{1,2,3\}$$
;
int a[3] = $\{7,24,9\}$

That is, 'int a[3]' defines the set of all integer arrays of size three.

Subtypes

Def: a <u>subtype</u> is a <u>subset</u> of the elements of a type.

Example: C The notation A < B means

A is a subtype of B.

Short is a subtype of int: short < int

Observations:

- (1) converting a value of a subtype to a values of the super-type is called <u>widening</u> type conversion. (safe)
- (2) converting a value of a supertype to a value of a subtype is called <u>narrowing</u> type conversion. (not safe)

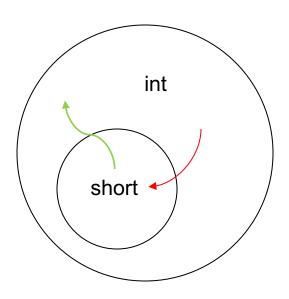
Example: C, partial type hierarchy

char < short < int < float < double

Subtypes give rise to type hierarchies and type hierarchies allow for automatic type coercion – widening conversions!

Subtypes

- A convenient way to visualize subtypes is using Venn diagrams
- Consider, short < int
- It is easy to see that the shorts are a subset of the integer values
- The green arrow represents a widening type conversion is always safe
- The red arrow represents a narrowing type conversion and is never safe



Why do we use types?

- Types allow the computer/language system to assist the developer write <u>better programs</u>.
 Type <u>mismatches</u> in a program usually indicate some sort of <u>programming error</u>.
 - Static type checking check the types of all statements and expressions at compile time.
 - <u>Dynamic type checking</u> check the types at runtime.

Type Equivalence

- Fundamental to type checking is the notion of type equivalence:
 - Figuring out whether two type description are equivalent or not
 - This is especially important for constructed types like class/struct objects.

Type Equivalence

I. <u>Name (nominal) Equivalence</u> – two objects are of the same type if and only if they share the same <u>type name</u>.

Example: Rust – constructed type

```
1  struct Type1 {x:i64, y:i64}
2  struct Type2 {x:i64, y:i64}
3
4  fn main () {
5    let x: Type1 = Type1{x:1,y:2};
6    let y: Type2 = x;
7    println!("{:?}",y);
8  }
Error; even the same,
```

Error; even though the types look the same, their names are different, therefore, Rust will not compile.

*Rust uses name equivalence

Type Equivalence

II. <u>Structural Equivalence</u> – two objects are of the same type if and only if they share the same <u>type structure</u>.

Example: Haskell

```
type Type1 = (Integer, Integer)
type Type2 = (Integer, Integer)

x :: Type1
y :: Type2

x = (1,2)
y = x
```

Even though the type names are different, Haskell correctly recognizes this statement.

Haskell uses <u>structural equivalence</u>.

- Type inference refers to the automatic detection of the data type of an expression in a programming language and to make sure that all expressions and statements are properly typed.
 - We often refer to this as "type checking" a program
- To see how this might work let's work through an example.

 Assume we have the following statements in a programming language like C:

```
int x = 3;
int y = (2 * x);
```

- We want to make sure that all the assignments are legal.
- We will use the type notation '3.integer' indicating that this syntactic unit has the type integer.

 We start at the primitives on the right side of the assignments of the first statement and then stepping through all the remaining statements

```
int x = 3;
int y = (2 * x);
```

int
$$x = 3$$
.integer;
int $y = (2 * x)$;

Start with the primitives on the right-hand side for the first statement

If we have evaluated a top-level entity, then check against left-hand side. If it type checks accept it, if not reject it. If you not at top-level keep inferencing.

```
int x = 3.integer;
int y = (2.integer * x.integer);
```

Process the next statement

```
int x = 3.integer;
int y = (2.integer * x.integer).integer;
```

If you not at top-level keep inferencing.

```
int x = 3.integer;
int y = (2.integer * x.integer).integer; ✓
```

Accept: we can assign an integer value to an integer variable.

If we have evaluated a top-level entity, then check against left-hand side. If it type checks accept it, if not reject it.

 Let's try a program with a bug in it. In C we have the hierarchy, short < int

```
int x = 3;
short y = (2 * x);
```

```
int x = 3.integer;
short y = (2^* x);
```

int
$$x = 3$$
.integer; \checkmark short $y = (2 * x)$;

If we have evaluated a top-level entity, then check against left-hand side. If it type checks accept it, if not reject it. If you not at top-level keep inferencing.

```
int x = 3.integer;
short y = (2.integer * x.integer);
```

```
int x = 3.integer;
short y = (2.integer * x.integer).integer;
```

```
int x = 3.integer;

short y = (2.integer * x.integer).integer; X
```

Reject: cannot assign a member of a supertype to a subtype.

Example: To see this check out the repl at https://replit.com/@lutzhamel/C-types#int.c

Type Inferencing in Asteroid

- Type inferencing for assignment statements works a little bit different in Asteroid:
 - The types must match exactly, no type conversion is supported during assignments
 - This is because Asteroid does not support a type hierarchy.

```
ast> let r:%real = 1.
error: pattern match failed: conditional pattern match failed
ast> let r:%real = toreal(1).
ast> let r:%real = 1 + 2.5.
error: found 'integer + real' expected 'real + real'
ast> let r:%real = toreal(1) + 2.5.
ast>
```

Let Q be the set of all negative integer values less than zero, $Q = \{-1,-2,-3,-4,-5,-6,-7,-8,-9,...\}$

- Let P be the set of all negative integer values evenly divisible by two, $P = \{-2, -4, -6, -8, ...\}$
- Then, is the following statement type safe assuming that x is declared as type Q and y is declared as type

$$x := (y+(-1))$$

where -1 is a member of type Q.

Hint: A type is a set of values!

Answer:

- First, we have to determine if there is a subtype-supertype relationship between Q and P. There is, because P is a subset of Q, P < Q
- Second, now we can do our type inferencing on the statement

$$x := (y+(-1))$$

$$x := (y.P+(-1).Q)$$

Start with primitives on rhs.

$$x := (y.Q+(-1).Q)$$

Both operands of + have to have the same type. We know that P < Q, therefore we can replace P with Q on the left operand to +.

$$x := (y.Q+(-1).Q).Q$$

If the input type to + is Q then the output type is also Q

$$x.Q := (y.Q+(-1).Q).Q$$

The variable x was declared as type Q. Therefore, we have an assignment of a Q value to a Q variable which is always safe.

What about the assignment,

$$y := (-1)$$

is it type safe?

$$y := (-1).Q$$

Start with primitives on rhs

Look at type on lhs. NOT type safe because P < Q. You cannot store a value from a supertype into a variable of a subtype.

Types & Objects

- In any OO language class definitions create new types
- Objects are the values in those types
- In OO languages that support inheritance, inheritance creates a subtype-supertype relationship in the class hierarchy

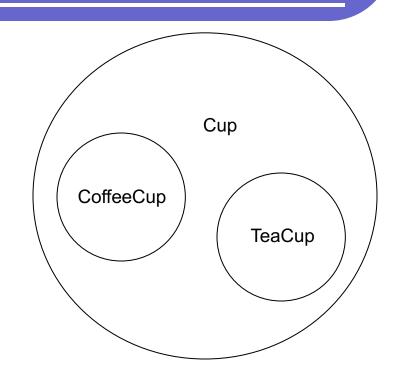
Types & Objects

Example: Java

```
class Cup { ... };
class CoffeeCup extends Cup { ... };
class TeaCup extends Cup { ... };
```

Which ones of the following statements are safe and which ones are not?

- 1. Cup x = new Cup();
- 2. Cup y = new CoffeeCup();
- TeaCup z = new Cup();
- TeaCup t = new TeaCup();
 Cup c = t;



CoffeeCup < Cup TeaCup < Cup

Note: Type coercion in type hierarchies gives rise to polymorphic programming in OO - objects can appear in different type contexts. More on that later.

Object-Oriented Programming

- Classic OO languages are based around inheritance hierarchies.
- The main distinguishing feature between them is whether they support single or multiple inheritance.
 - C++ and Python support multiple inheritance
 - Java supports single inheritance
- There are three main problems with inheritance-based OO languages.

Problem #1

- Bloated method inheritance that is, each child in an inheritance hierarchy will inherit ALL of the methods of its ancestors.
- This is true for both single and multiple inheritance.

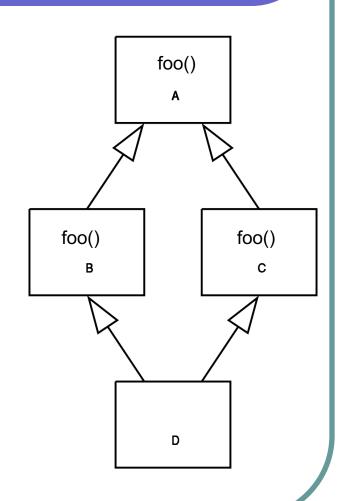
Problem #2

- The diamond problem sometimes referred to as the 'deadly diamond of death'
- This occurs in languages with multiple inheritance

The Diamond Problem

Briefly:

- An ambiguity that arises when two classes B and C inherit from A, and class D inherits from both B and C.
- If there is a method in A that B and C have overridden, and D does not override it, then which version of the method does D inherit: that of B, or that of C?
- That is: D.foo() which foo() should be called?
- This gets really problematic in deep inheritance structures.



The Diamond Problem

- Different languages deal with the diamond problem in different ways
 - C++ uses a fully qualified syntax
 - Python uses a class hierarchy linearization algorithm (C3 linearization or MRO) to resolve ambiguities

```
CSC301-git — python3 — 117×21
(base) MacBook:CSC301-git lutz$ python3
Python 3.7.6 (default, Jan 8 2020, 13:42:34)
[Clang 4.0.1 (tags/RELEASE_401/final)] :: Anaconda, Inc. on darwin
Type "help", "copyright", "credits" or "license" for more information.
>>> class A:
        pass
>>> class B(A):
        pass
>>> class C(A):
        pass
>>> class D(B.C):
        pass
>>> D.mro()
[<class '__main__.D'>, <class '__main__.B'>, <class '__main__.C'>, <class '__main__.A'>, <class 'object'>]
>>>
```

Problem #3

- A third problem that frequently arises in inheritance-based OO languages are rigid class structures
 - This usually manifests itself in class hierarchies that are difficult to evolve in face of changing software requirements

Object-Based Programming

- A response to these problems is that recent languages no longer support inheritance and are object-based
- Of the three new big languages, Rust, Go, and Swift, only Swift supports a full OO model.
- Asteroid is object-based, that is, it supports objects but not inheritance.

Take Away

- Types are sets of values, typically with a common representation and common set of operations.
- Types in programming languages allows compilers and interpreters to check for consistency in your programs.
- Inconsistencies/bugs usually show up a type mismatches.
- Type equivalence between constructed types can be established in one of two ways, name equivalence or structural equivalence.
- Class hierarchies in OO languages give rise to subtype-supertype relationships due to inheritance.

Exercise

- Let P={1,2,3,4,5} and Q={2,4}
- Let x:P and y:Q, determine if the following are legal:
 - 1. x = (1+1) with 1.P
 - x = (4+2) with 4.P and 2.P
 - 3. x = (1+y) with 1.P

Assignments

- Reading: MPL Chap 6
- Assignment #2 See BrightSpace